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Coherent Anti-Stokes Raman Spectroscopy Temperature Measurements in a Hydrogen-Fueled Supersonic Combustor

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Coherent anti-Stokes Raman spectroscopy (CARS) thermometry has been used to obtain static temperature cross sections in a three-dimensional supersonic combustor flowfield. Data were obtained in three spanwise planes downstream of a single normal fuel injector which was located downstream of a rearward-facing step. The freestream flow was nominally Mach 2 and was combustion heated to a total temperature of 1440 K (yielding a static temperature of about 800 K in the freestream) to simulate the inflow to a combustor operating at a flight Mach number of about 5.4. Since a broadband probe laser was used an instantaneous temperature sample was obtained with each laser shot at a repetition rate of 10 Hz. Thus root-mean-square (rms) temperatures and temperature probability density functions (pdf's) were obtained in addition to mean temperatures.

I. Introduction

COHERENT anti-Stokes Raman spectroscopy (CARS) is a widely accepted nonintrusive laser diagnostic technique capable of measuring the temperature and concentration of various species. In this experiment only one species, nitrogen, was investigated. Nitrogen was selected because it is a major constituent of the flow (78% of nonvitiated air), because it produces a large CARS signal, and because that signal is relatively simple to interpret. Only CARS temperature measurements were made in this test; nitrogen concentration measurements were not attempted although they are planned for the next phase of the project.

A CARS signal is generated by the interaction of a laser beam of arbitrary wavelength called the "pump" beam and a second beam termed the "Stokes" beam. The frequency of the Stokes beam must be chosen such that the frequency difference between the Stokes beam and the pump beam matches a Raman resonance of the molecule being investigated. When the pump and Stokes beams are properly overlapped in the measurement volume an anti-Stokes beam is generated with an intensity that can be related to the population of molecules in that volume which has a particular combination of rotational and vibrational energy. Information about a single rotational/vibrational energy level is insufficient to determine the temperature of the volume so a number of different energy levels must be probed. This can be done by scanning the frequency of the Stokes beam or by generating a number of frequencies all at once. In the second approach called "broadband" CARS, a Stokes beams containing a band of wavelengths is created. Each frequency component of the Stokes beam produces an anti-Stokes signal which can be related to the population of a different energy level. The temperature of the volume can then be derived from the measured distribution of energy level populations.

A tremendous advantage of CARS is that the signal is coherent—it comes out as an anti-Stokes beam. Compare this to spontaneous Raman scattering in which the signal is distributed uniformly in all directions and must be collected with large aperture lenses. Obtaining optical access in supersonic combustors is a nontrivial matter and the ability to collect light through small apertures greatly simplifies the task. The primary advantage of broadband CARS is that it provides sufficient information to determine temperature in a single laser shot. This capability is required for the investigation of turbulent flows since they are, by definition, unsteady.

Previous work at Langley has employed CARS in supersonic reacting flows. Nitrogen temperatures and nitrogen and oxygen concentrations were obtained by Jarrett et al.[1] in a supersonic open jet laboratory burner. Other workers have successfully used CARS in ducted scramjet combustors. Anderson and Eckbreth,[2] e.g., used CARS to measure nitrogen temperature, as well as water, hydrogen, and nitrogen concentration along a one-dimensional profile in a supersonic, hydrogen-fueled, combustor.

The contribution of the present work is twofold. First it represents the birth of a new measurement capability in the relatively large ducted test facilities at Langley. Since each facility presents its own set of optical access restrictions as well as vibration, acoustic, and other environmental challenges, the acquisition of useful data in any new facility can be considered a milestone. Second, the use of a two-dimensional measurement volume translation system has allowed the collection of a more extensive data set than has been previously reported elsewhere. Here, three streamwise planes of data are presented rather than a single profile or even a single plane. This information dramatically enhances the ability of the reacting flow researcher to interpret fluid, mechanical, and combustion processes.

Flow over a rearward-facing step was chosen for the test since it is something of a canonical scramjet combustor. The freestream flow was nominally Mach 2 and was combustion

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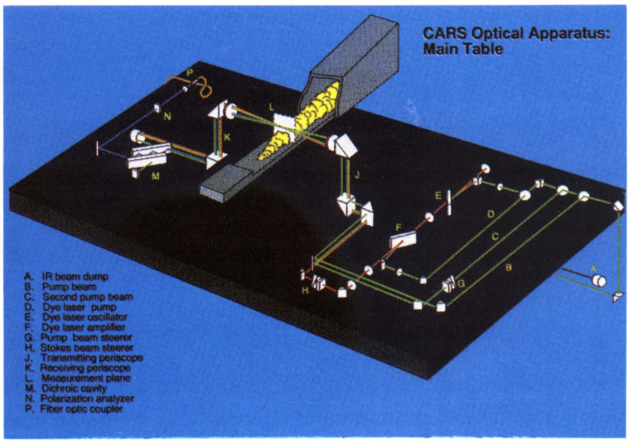

Fig. 1 Diagram of CARS apparatus primary optical table.

heated with hydrogen to simulate the inflow to a combustor operating at a flight Mach number of about 5.4. Hydrogen fuel was injected through a single port, normal to the free-stream flow and downstream of the step. Measurements were taken in spanwise planes at three different stations downstream of the injector.

II. Experimental Details

A. Optical Apparatus

Optical components were mounted on both the main table in the test cell and on a second, smaller table, in the control room. Figure 1 is an overview of the optical components on the main table. The CARS system was driven by a frequency-doubled Nd:YAG laser (Quanta-Ray DCR-1A) which produced 10 ns pulses at 532 nm (green) at a repetition rate of 10 Hz. The green beam (~185 mJ/pulse) was split into three beams of approximately equal power. Two were used as pump beams and the third drove an axially-pumped broadband dye laser. Rhodamine 640 in methanol was used in the dye laser to generate a broadband Stokes beam centered around 607 nm (red), the frequency range which is required to probe nitrogen. The dye concentrations in the oscillator and amplifier were controlled separately, the oscillator concentration was varied to adjust frequency and the amplifier concentration was varied to maximize intensity.

At the measurement volume the two pump beams and the Stokes beam were crossed in a planar BOXCARS configuration (see Fig. 2). With planar BOXCARS a complete spanwise survey plane can be accessed with no more than a pair of slits in the tunnel walls, thus minimizing optical access requirements (but with some added optical complexity as described below). This is in contrast with folded BOXCARS which requires a slit several times wider than a single beam to allow for the crossing angle of the pumps and probe. "Used" CARS was also a candidate for this experiment since the DCR1-A had a doughnut shape due to its unstable resonator. However, for the relatively high *f* number field lenses employed in this test it would have provided considerably lower spatial resolution in the optical pathwise direction.

The optical access slits were sealed with ½-in.-thick Pyrex® windows bolted on the outside of the tunnel so that the glass was recessed from the inner wall and the hot flow (by about 2 in.). The cavities between the flow and the windows was purged with shop air to keep contaminants off the windows. Pneumatically-actuated rotary shutters were installed in the space between the windows and the flow for protection during startup and shutdown, but were found to be unnecessary. To provide for the motion of the measurement volume within the plane defined by the slits in the tunnel walls, the beam-steering prisms and primary lenses were mounted on a computer-controlled, stepping-motor-driven, two degree-of-freedom translation system (see Fig. 3).

In this implementation of planar BOXCARS, the Stokes beam partially overlapped one of the pump beams so these two beams had to be combined using an appropriate dichroic mirror. On the receiving side, the anti-Stokes beam partially overlapped the other pump beam. Since the blue anti-Stokes beam which contained the signal was very weak compared to the green pump beam, it was separated from the pump using a multipass dichroic cavity (as was done by Fujii et al.[3]). Each time the green and blue beams struck a cavity mirror 98% of the blue was reflected and 90% of the green was passed, thus separating the signal from the interference. The added complexity of dichroic combining and splitting optics is the penalty paid for the use of planar BOXCARS. With folded BOXCARS these optics are not required since the probe and signal beams are spatially separated from the pump beams.

Having been isolated, the blue signal beam was focused on the face of a 50-μm multimode fiber optic. The fiber, 20-m long, transmitted the signal to a spectrometer which was located remote to the main table in the test cell control room. Some additional optics were required to condition the signal beam before it entered the spectrometer (see Fig. 4 for a diagram of these conditioning optics). First, the beam was expanded using a 14.8-mm focal length microscope objective. Then it was shaped with a pair of cylindrical lenses so that it matched the tall thin pixels (2500 × 25 μm) on the one-dimensional intensified photo-diode array (IPDA) at the back plane of the spectrometer. This IPDA was a Peltier-cooled EG+G PARC model 1420 detector with a P-20 phosphor. Because of the large temperature fluctuations in the turbulent flow being studied, a splitter plate had to be used to provide the system with more dynamic range than is intrinsic to the IPDA. Dynamic range enhancement was implemented with a version of the selectively-coated reflector described by Eckbreth.[4] It generated two channels, one attenuated by about an order of magnitude, side-by-side at the entrance plane of the spectrometer. In practice, the high temperature shots were recorded on the 90% *T* channel (*T* = transmittance) and the lower temperature larger intensity shots were recorded on the 9% *T* channel since these shots saturated on the 90% *T* channel.

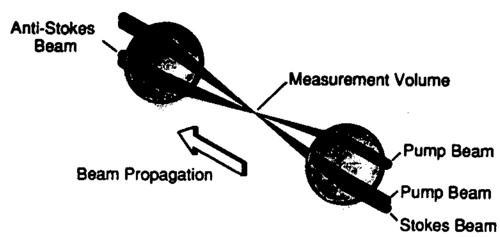

Fig. 2 Arrangement of beams for planar BOXCARS.

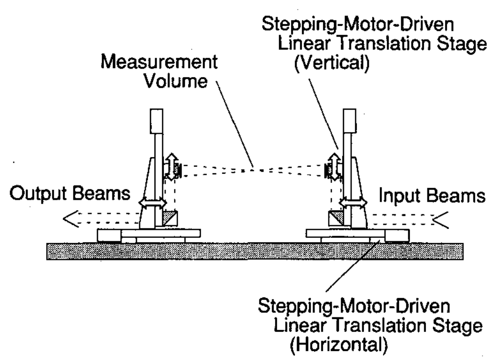

Fig. 3 Two degree-of-freedom translation system used to manipulate the measurement volume.

An additional optic was added after the system had been operating for some time. It was discovered that the combination of focal length and power used in the system was sufficient to cause breakdown of the air in the measurement volume at room temperature. (Breakdown did not occur at the high temperatures of interest found in the combustor flow.) When breakdown occurred a base level of background signal was observed in the spectrum. This base level was found to be linearly polarized with an orientation at 90 deg to the vertically polarized CARS signal. A polarization analyzer was then installed to reject this background. Careful examination of the CARS signal at flame temperatures showed a much smaller base level that was similarly attenuated by the polarization analyzer. Thus, all data presented in this article were acquired with polarization rejection of this small, but unidentified background.

B. Test Cell

Located in the test cell illustrated in Fig. 5 is a combustion-heated plenum chamber used to generate the high enthalpy flows required to test scramjet combustor configurations. In this facility hydrogen is added to the plenum and burned to heat the air while simultaneously, pure oxygen is added to maintain the oxygen content of the heated air at 21% by mole. (The heated "air" contains a significant fraction of water and much smaller fractions of other combustion products, all roughly in proportion to the amount of hydrogen that is added.) Total temperatures as high as 2200 K can be achieved but the cases reported in this article were taken at T_0 = 1440 K. For the present work, this high enthalpy vitiated air was expanded through a two-dimensional Mach 2 nozzle to a static temperature of about 800 K. This flow, which was then fed to the combustor model, simulated the flow out the back of an isentropic inlet operating at a flight Mach number of about 5.4. At the downstream end of the combustor model an annular supersonic air ejector was used to reduce the back pressure on the duct which kept the flow supersonic to the end of the model.

Conditions in the test cell were not conducive to laser diagnostics so considerable effort was devoted to "hardening" the CARS system for the environment. Vibration of optical components could come from either the floor through the table legs, or directly through acoustic excitation driven by tunnel noise. To compensate, only the heaviest duty mounts with the strongest springs and/or locking mechanisms could be used. All optics were glued and/or screwed to their mounts. As an additional barrier to acoustic excitation the entire CARS table was shrouded in a sheet metal box. To meet test cell safety requirements this box also was purged with shop air to keep any possible hydrogen leaks away from the ignition sources in the laser system. Table vibrations were reduced by mounting the entire apparatus on rubber blocks. Air suspension legs were available but were found to be unnecessary (and unreliable as far as table positioning was concerned).

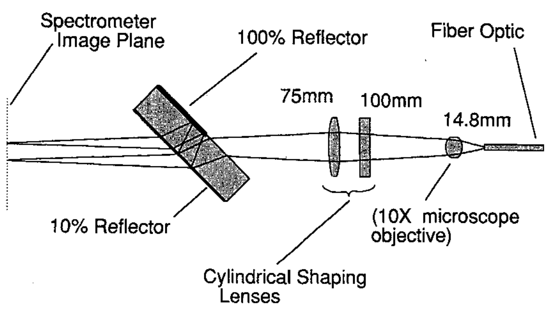

Fig. 4 Signal-conditioning optics at the entrance to the spectrometer.

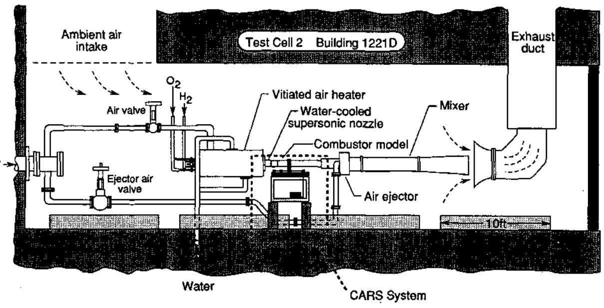

Fig. 5 Overview of the combustion test cell.

III. Data

A. Practical Considerations

Test cell no. 2 is a short duration blowdown facility. Normally runs last about 10 s if fuel is added to the model and as long as 35 s if fuel is not added. For the present hardware, longer run times produced damage to the uncooled model's surfaces through melting and/or thermal stress and cracking. Therefore, for single-point CARS measurements many runs were required to map out a number of grid points in a data plane. Typically a data-taking session consisted of 10–40 runs between 8–20 min apart and included about 45 min of facility startup after the test cell door was closed and about 30 min of shutdown before the door could be opened. During the session access to the test cell was impossible for safety reasons due to the presence of pressurized oxygen and hydrogen lines. Therefore, any adjustments required to keep the CARS system operating had to be done remotely from the control room.

Overlap of the pump and Stokes beams in the measurement volume is the most critical alignment in a CARS system. To allow remote adjustment two beam steerers were installed, one was installed on one of the two pump beams and the other on the Stokes beam. Each steerer consisted of a pair of 2 deg optical wedges mounted in a pair of stepping-motor-driven rotary stages. Installed in series along the optical path (see items *G* and *H* in Fig. 1), the wedges could be rotated independently about the axis of a beam, thus controlling its deflection and its position relative to the other beams in the measurement volume. This positioning system was found to be both flexible (deflections from microns to millimeters were possible with 2 deg wedges) and repeatable.

Drift in beam overlap during the course of a running session was frequent, particularly at the beginning of a session, and was most likely due to thermally induced deformations of the optical table. Table temperature was affected by two different sources. One was the radiant heat from the tunnel. As a countermeasure, foam insulation was installed on the table surfaces exposed to radiation from the tunnel. The second and likely more damaging source, was the exposure of the table to the outside temperature once the test cell doors were closed. As part of the facility safety procedures, outside air was vented through the test cell during each run (see Fig. 5) exposing the table to dramatic temperature swings. In addition, again as a safety precaution, electrical power to the test cell climate control system was cut off during running sessions, thus aggravating the situation.

Although remotely readjusting beam overlap greatly improved signal levels the initial signal levels were often not recovered. One possibility is that the adjustment of the beam overlap moved the measurement volume slightly. Therefore, signal levels could not be recovered without then adjusting the receiving optics accordingly. Remote adjustments for the fiber collection system are under consideration for the next phase of testing.

B. Data Reduction

Temperatures were derived by comparing each sampled CARS spectrum to a library of calculated spectra which spanned the anticipated temperature range. This library of spectra, its elements separated by 20 K increments, was generated using the technique of Antcliff and Jarrett.[5] A temperature match was found by locating the library entry with the minimum mean square error relative to the measured spectrum (both spectra were normalized by their peak intensities prior to the comparison).

The only departure from the library parameters used by Antcliff and Jarrett was in the instrument function. This function represents the transfer function of the optical system and accounts for the smearing of the sampled spectra due to the limits of the imaging optics and the IPDA. In a perfect optical system the instrument function is a delta function; for the present system the instrument function was modeled as a Lorentzian because it produced the most acceptable fit between the calculated spectra and the data. Its width was selected by forcing the room temperature library entry to match the sampled room temperature CARS spectrum as closely as possible.

An advantage of the fiber-coupled spectrometer arrangement employed in this CARS system was that the adjustments affecting the instrument function were isolated from the test cell environment. This reduced the possibility that the instrument function might change during a run as a result of vibrations or thermal deformations of the hardware. Also, since the fiber acted as a signal buffer between the CARS table and the spectrometer, modifications to the system (repositioning or realignment of the optics for instance) could be made without having to readjust the instrument function.

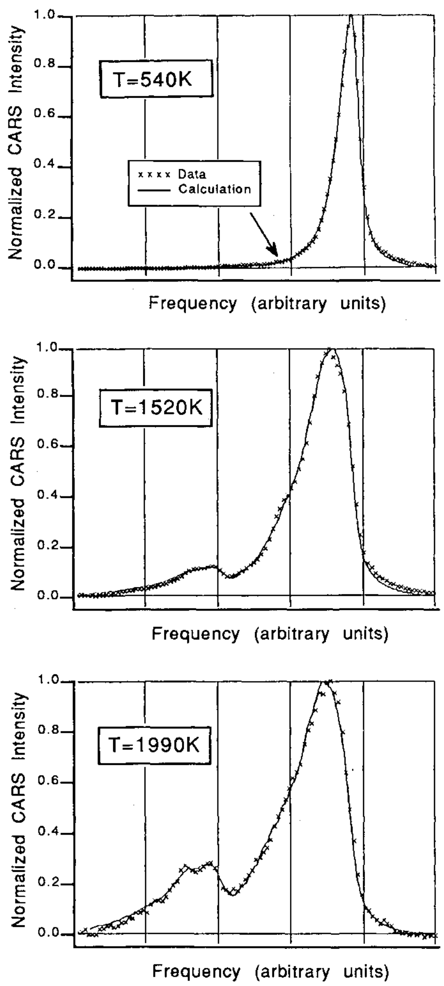

Fig. 6 Comparison of measured single-shot CARS spectra and calculated CARS spectra over the temperature range encountered in the combustor flow.

Sample fits for single-shot data taken over a range of temperatures representative of those encountered in the test cell flow are shown in Fig. 6. These shots were selected from the data base presented below in the "Results" section.

C. Results

Data were taken in three spanwise planes. Station 1 was located 12.3-in. downstream of the rearward-facing step, station 2, 27.3 in., and station 3, 42.9 in. (That translates into 49.2, 98.2, and 171.7 step-heights downstream of the rearward-facing step or into 37.4, 86.4, and 149.1 injector diameters downstream of the fuel injector.) The step was 0.3-in. high and the fuel injector was located 10 step-heights downstream of the step. Just upstream of the step, the duct was 3.46-in. wide by 1.5-in. high. On the fuel side of the duct, one wall diverged at 3 deg beginning 0.35-in. downstream of the fuel injector (see Fig. 7).

Data were taken with and without fuel injection at a heater total temperature of 1440 K and a total pressure of 120 psia. The mean static pressures (measured at the wall) ranged from a high of about 28 psia (1.9 atm) at station 1, to a low of about 12 psia (0.8 atm) at station 3. Graphical results for the case with fuel addition are presented in Fig. 8. Here, each plane represents a grid of 9 × 6 data locations with from 20 to several hundred data samples at each location. All three data planes are 3-in. wide and increase in height from 1.7 in. at station 1 to 3.2 in. at station 3. Heater data (no fuel addition) were taken at stations 2 and 3. Station 3 heater data are presented graphically in Fig. 9.

Additional CARS data were taken in a cold nitrogen jet to determine the effective size of the measurement volume in the optical pathwise dimension. Using the translation system, the measurement volume was traversed through a thin (~0.2 mm), planar jet of nitrogen surrounded by a coflowing argon bath. A plot of CARS signal intensity as a function of distance through the planar jet is presented in Fig. 10 and demonstrates a FWHM (full width, half maximum) spatial resolution of about 0.7 mm (0.027 in.). The spatial resolution in the cross-beam direction is much finer; in the diffraction-limited case it is approximately 43 μm.

IV. Discussion

The heater data taken at stations 2 and 3 were normalized to a total temperature of 1440 K; the actual total temperature was 1440 ± 50 K with the variations occurring during runs as well as from one run to the next. The normalized data show fairly uniform temperature cross sections at both stations. At station 2 the mean temperature averaged over the cross section was 617 K with an rms over the mean cross section of 32 K (5.2%), and at station 3 the mean was 536 K with an rms of 26 K (4.9%). The mean temperature data at station 3 are plotted with an exaggerated scale in Fig. 9, and with this heightened sensitivity some mean structure is evident. Low temperature regions on either side of the centerline may indicate the presence of counter-rotating streamwise "rollers" (weak vortices) in the central region of the duct.

For comparison with the mean CARS data, static temperatures were calculated with the one-dimensional code of Gordon and McBride.[6] This code accounts for the variation of γ (the ratio of specific heats) with temperature, although it does

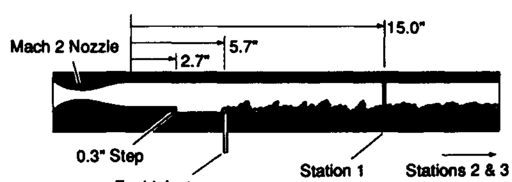

Fig. 7 Scramjet combustor model geometry.

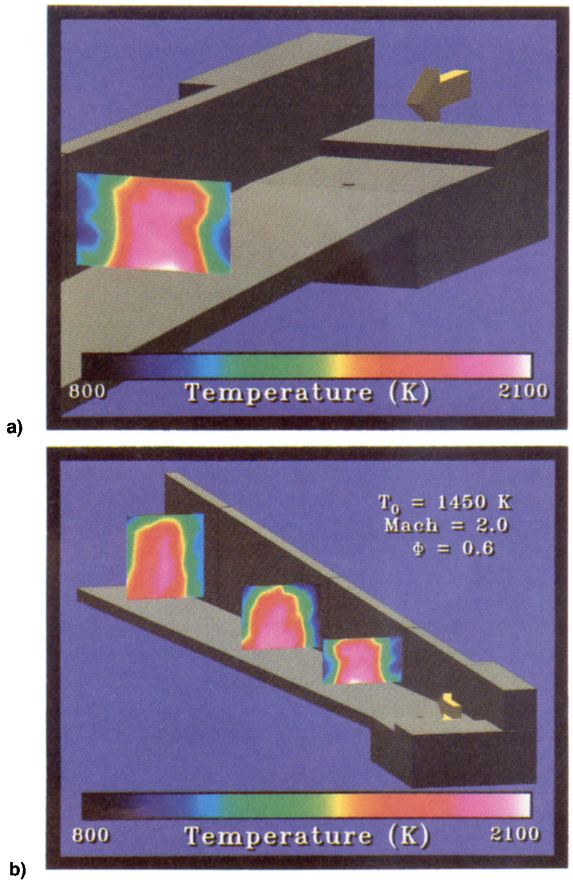

Fig. 8 Summary of mean temperature data surveys: a) close-up of model and station 1 data, and b) all three data stations.

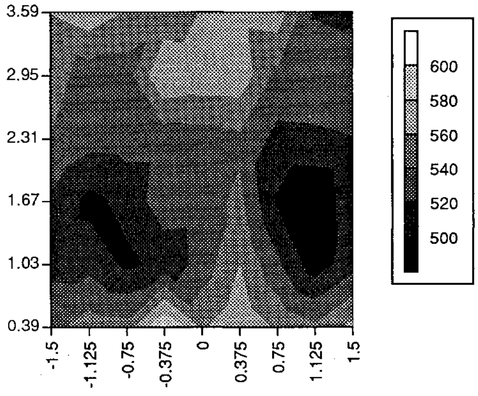

Fig. 9 Mean temperature data without fuel addition (heater only) at station 3. Temperature scale is exaggerated for clarity.

not account for heat lost by the flow to the water-cooled nozzle and to the duct. The measured values were 9 and 11% lower than the corresponding calculated values at stations 2 and 3, (617 vs 689 K, and 536 vs 606 K). This discrepancy is likely due to the unaccounted heat loss, or perhaps a systematic error in the calibration of the CARS system, or a combination of both.

The static temperature data for the fueled model (Fig. 8) suggest a hot, slowly spreading jet attached to the fueled wall and defined by fairly sharp borders. As might be expected at a fuel equivalence ratio of only $\Phi = 0.6$, the jet does not fill the duct; its boundaries are still distinct at station 3 (the farthest downstream station).

Recall that CARS thermometry provides an odd sort of temperature measurement, the temperature of a single species—in this case nitrogen. All the data records used to gen-

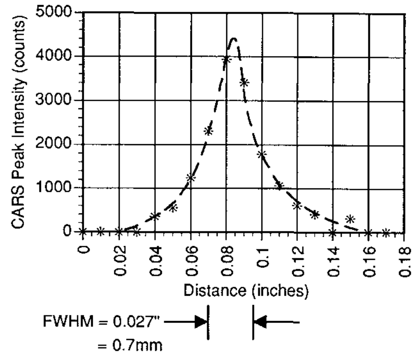

Fig. 10 Results of spatial resolution test of CARS system.

erate Fig. 8 had successful shot yields of 95% or better, i.e., very few shots produced no signal. This demonstrates that everywhere measurements were taken there was enough nitrogen to get a CARS signal, even at station 1 where one would expect the least amount of mixing. The contents of the jet, presumably water vapor and lesser amounts of other hydrogen/oxygen combustion products, as well as the measured hot nitrogen that must have been mixed in by the entrainment of freestream air, suggest that fairly thorough mixing has occurred upstream of station 1. However, this jet with its well-mixed interior does not exhibit much growth as it travels downstream.

Contrast this notion of a "well-mixed" jet to one in which the combustion occurs over a longer streamwise extent along the borders of a fuel jet which retains a hydrogen core far downstream. In this case the CARS signal would disappear in the hydrogen core and show high temperatures along the reacting borders. The combustion would take place along the slowly growing free shear layer shrouding the hydrogen core.

Instantaneously, the contents of the well-mixed jet appear to be even more homogeneous than they seem in the mean. Fig. 11a is a mean one-dimensional temperature profile taken from the two-dimensional data set for station 1 (a one-dimensional profile taken parallel to the tunnel floor at about half the height of the jet). It represents a traverse from one side of the jet to the other. In the mean, the static temperature gradually rises from about 1100 K at the edge of the jet through a smooth transition to a plateau of near 2000 K in the middle of the jet, then back down through another smooth transition. However, in Fig. 11b (the rms temperature plot for the same profile), there are two peaks corresponding to these transition regions indicating relatively large shot-to-shot temperature fluctuations there. Probability density functions of temperature along the same profile (shown in Fig. 12) reveal the cause. In the middle part of the jet (Fig. 12, $x = 0.75$ through -0.375) the temperatures are centered around 2000 K. Outside the jet ($x = 1.5$, -1.125, and -1.5) the temperatures are centered around 1100 K. However, at the edges of the jet (Fig. 12, $x = 1.125$ and $x = -0.75$) the pdf's are bimodal, showing temperatures grouped around both 1100 and 2000 K, the large moment thus producing a large rms. Although the mean temperature in the transition zone at the edge of the jet is around 1500 K almost no samples were actually at that value. The conclusion must be that the jet is a fairly uniform mass of 1900–2000 K fluid with instantaneously ragged borders which dart in and out of the measurement volume as the jet travels downstream. This averaging produces the illusion of smooth borders and makes the well-mixed interior appear less homogeneous than a "snapshot" of the flow would reveal.

An obvious, but important conclusion to be drawn from finding a well-mixed jet full of combustion products at station 1 is that much of the mixing and combustion has taken place upstream of that point. (Only hot nitrogen was actually measured with CARS, but its presence at a high temperature in

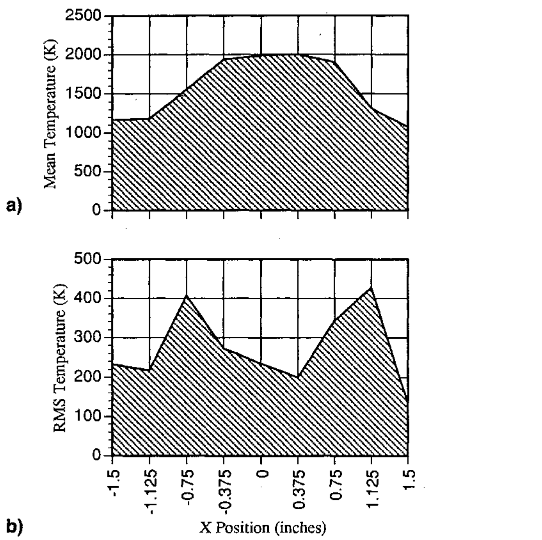

Fig. 11 a) Mean temperature profile along $y = 0.56$ in station 1 data with fuel addition, and b) rms temperatures along the same profile.

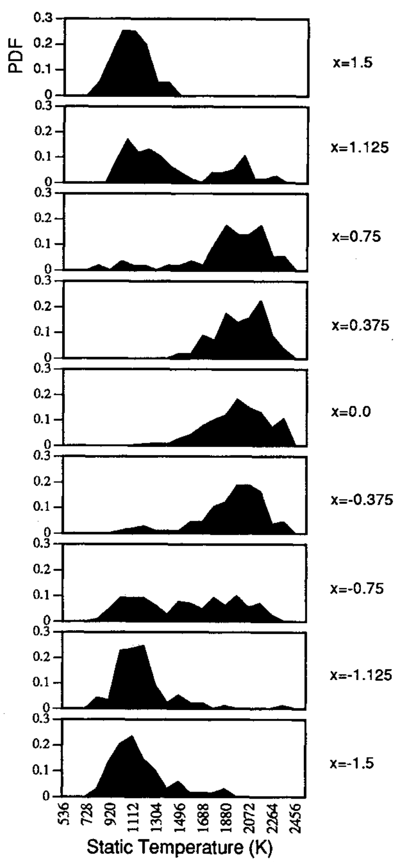

Fig. 12 Probability density functions of temperature along $y = 0.56$ profile of station 1 data with fuel addition.

nearly every sample suggests an advanced state of combustion.) One explanation is that the fuel jet is severely shredded by the strong perturbations experienced behind the rearward-facing step and around the injection port. Considerable mixing and combustion takes place in the first 10 or 20 step-heights downstream of injection, after which the jet of hot products interacts with the freestream flow with something much more like a free shear layer growth rate. If this hypothesis is true, more emphasis needs to be placed on understanding the "front ends" of supersonic mixing interactions to complement the considerable amount of effort that has been expended on understanding the streamwise development of supersonic equilibrium free shear layers.

V. Summary

CARS measurements of static temperature downstream of a fuel injector which is itself downstream of a rearward-facing step, reveal a nearly uniform jet of hot combustion products that retains its identity well downstream. Instantaneous measurements suggest that the supersonic jet is quite ragged and that its interior is an even more homogeneous mix of combustion products than the mean data suggest. The success of this experiment demonstrates the ability of this "hardened" CARS system to function remotely in the noise, vibrations, and ambient temperature swings of the combustion test cell environment.

Acknowledgments

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